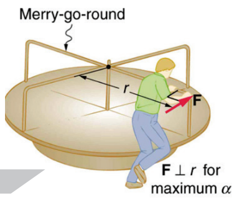


Unit - 5

Rotational Motion

5.1 Introduction

Rigid body : A rigid body is a body that can rotate with all the parts locked together and without any change in its shape.

5.2 Centre of Mass

Centre of mass of a system is a point that moves as though all the mass were concentrated there and all external forces were applied there.

(1) Position vector of centre of mass for *n* particle system : If a system consists of n particles of masses m_1, m_2, m_3 .... m_n, whose positions vectors are $\vec{r_1}, \vec{r_2}, \vec{r_3}, \ldots\ldots\ldots \vec{r_n}$ respectively then position vector of centre of mass

$$\vec{r} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2} + m_3 \vec{r_3} + \ldots\ldots\ldots m_n \vec{r_n}}{m_1 + m_2 + m_3 + \ldots\ldots\ldots m_n}$$

If two masses are equal *i.e.*, $m_1 = m_2$, then position vector of centre of mass $\vec{r} = \frac{\vec{r_1} + \vec{r_2}}{2}$.

(2) Important points about centre of mass

(i) The position of centre of mass is independent of the co-ordinate system chosen.

(ii) The position of centre of mass depends upon the shape of the body and distribution of mass.

(iii) In symmetrical bodies in which the distribution of mass is homogenous, the centre of mass coincides with the geometrical centre or centre of symmetry of the body. Centre of mass of cone or pyramid lies on the axis of the cone at point distance $\frac{3h}{4}$ from the vertex where h is the height of cone.

(iv) The centre of mass changes its position only under the translatory motion. There is no effect of rotatory motion on centre of mass of the body.

(v) If the origin is at the centre of mass, then the sum of the moments of the masses of the system about the centre of mass is zero *i.e.,*

$$\Sigma m_i \vec{r_i} = 0.$$

(vi) If a system of particles of masses m_1, m_2, m_3, ... move with velocity v_1, v_2, v_3, ... then the velocity of centre of mass

$$v_{cm} = \frac{\Sigma m_i v_i}{\Sigma m_i}$$

(vii) If a system of particles of masses m_1, m_2, m_3, ... move with accelerations a_1, a_2, a_3, ... then the acceleration of centre of mass

$$A_{cm} = \frac{\Sigma m_i a_i}{\Sigma m_i}$$

(viii) If $\vec{r}$ is a position vector of centre of mass of a system then velocity of centre of mass $\vec{v}_{cm} = \frac{d\vec{r}}{dt}$.

(ix) Acceleration of centre of mass $\vec{A}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \frac{d^2\vec{r}}{dt^2}$.

(x) Force on a rigid body $\vec{F} = M\vec{A}_{cm} = M\frac{d^2\vec{r}}{dt^2}$.

(xi) For an isolated system external force on the body is zero

$$\vec{F} = M\frac{d}{dt}(\vec{v}_{cm}) = 0$$

$$\Rightarrow \quad \vec{v}_{cm} = \text{constant.}$$

i.e., centre of mass of an isolated system moves with uniform velocity along a straight-line path.

5.6 Equations of Linear Motion and Rotational Motion

Rotational Motion
If angular acceleration is 0, ω = constant and $\theta = \omega t$
If angular acceleration α = constant then (i) $\theta = \frac{(\omega_1 + \omega_2)}{2} t$ (ii) $\alpha = \frac{\omega_2 - \omega_1}{t}$ (iii) $\omega_2 = \omega_1 + \alpha t$ (iv) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ (v) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ (vi) $\theta_{nth} = \omega_1 + (2n-1)\frac{\alpha}{2}$
If acceleration is not constant, the above equation will not be applicable. In this case (i) $\omega = \frac{d\theta}{dt}$ (ii) $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ (iii) $\omega d\omega = \alpha d\theta$

5.7 Moment of Inertia

Moment of inertia plays the same role in rotational motion as mass plays in linear motion. It is the property of a body due to which it opposes any change in its state of rest or of uniform rotation.

(1) Moment of inertia of a particle $I = mr^2$; where r is the perpendicular distance of particle from rotational axis.

(2) Moment of inertia of a body made up of number of particles (discrete distribution)

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots\dots$$

(3) Moment of inertia of a continuous distribution of mass, $dI = dmr^2$ *i.e.,*

$$I = \int r^2 dm$$

(4) Dimension : $[ML^2T^0]$

(5) S.I. unit : kgm^2.

(6) Moment of inertia depends on mass, distribution of mass and on the position of axis of rotation.

(7) Moment of inertia is a tensor quantity.

5.8 Radius of Gyration

Radius of gyration of a body about a given axis is the perpendicular distance of a point from the axis, where if whole mass of the body were concentrated, the body shall have the same moment of inertia as it has with the actual distribution of mass.

When square of radius of gyration is multiplied with the mass of the body gives the moment of inertia of the body about the given axis.

$$I = Mk^2 \text{ or } k = \sqrt{\frac{I}{M}}.$$

Here k is called radius of gyration.

$$k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \ldots\ldots r_n^2}{n}}$$

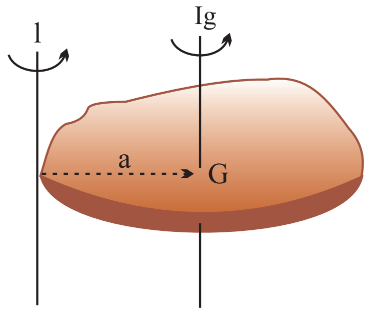
Note :

- For a given body inertia is constant whereas moment of inertia is variable.

5.9 Theorem of Parallel Axes

Moment of inertia of a body about a given axis I is equal to the sum of moment of inertia of the body about an axis parallel to given axis and passing through centre of mass of the body I_g and Ma^2 where M is the mass of the body and a is the perpendicular distance between the two axes.

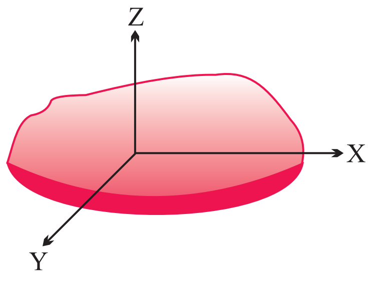
$$I = I_g + Ma^2$$



5.10 Theorem of Perpendicular Axes

According to this theorem the sum of moment of inertia of a plane lamina about two mutually perpendicular axes lying in its plane is equal to its moment of inertia about an axis perpendicular to the plane of lamina and passing through the point of intersection of first two axis.

$$I_z = I_x + I_y$$



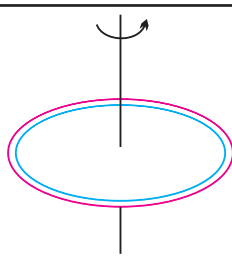
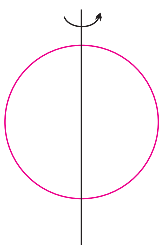
Note :

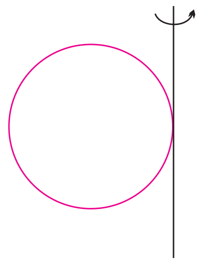
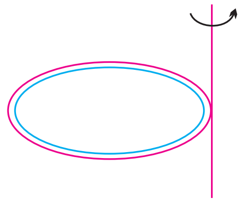
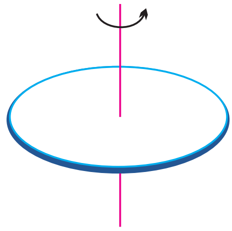
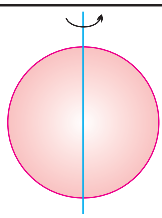
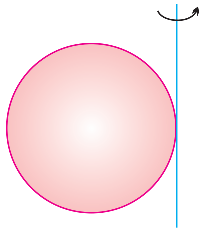
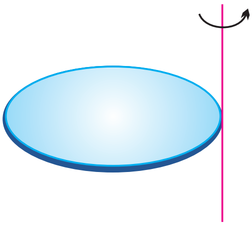
- In case of symmetrical two-dimensional bodies as moment of inertia for all axes passing through the centre of mass and in the plane of body will be same so the two axes in the plane of body need not be perpendicular to each other.

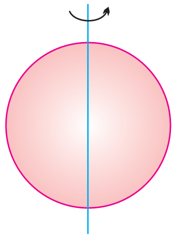
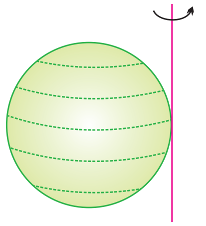
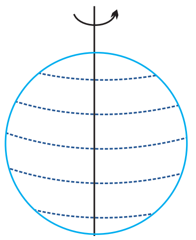
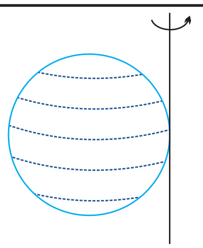
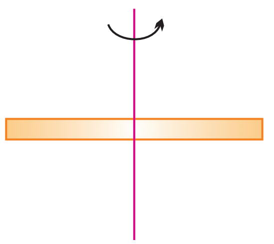
5.12 Analogy between Translatory Motion and Rotational Motion

Translatory motion		Rotatory motion	
Mass	(m)	Moment of Inertia	(I)
Linear Momentum	$P = mv$ $P = \sqrt{2mE}$	Angular Momentum	$L = I\omega$ $L = \sqrt{2IE}$
Force	$F = ma$	Torque	$\tau = I\alpha$
Kinetic energy	$E = \frac{1}{2}mv^2$ $E = \frac{P^2}{2m}$		$E = \frac{1}{2}I\omega^2$ $E = \frac{L^2}{2I}$

5.13 Moment of Inertia of Some Standard Bodies and Different Axes

Body	Axis of Rotation	Figure	Moment of inertia	K	K^2/R^2
Ring (Cylindrical Shell)	About an axis Passing through C.G. and perpendicular to its plane		MR^2	R	1
Ring	About its diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$

Body	Axis of Rotation	Figure	Moment of inertia	K	K^2/R^2
Ring	About a tangential axis in its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Ring	About a tangential axis perpendicular to its own plane		$2MR^2$	$\sqrt{2}R$	2
Disc (Solid cylinder)	About an axis Passing through C.G. and perpendicular to its plane		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Disc	About its diameter		$\frac{1}{4}MR^2$	$\frac{R}{2}$	$\frac{1}{4}$
Disc	About a tangential axis in its own plane		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$	$\frac{5}{4}$
Disc	About a tangential axis perpendicular to its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$

Body	Axis of Rotation	Figure	Moment of inertia	K	K^2/R^2
Solid Sphere	About its diametric axis		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$	$\frac{2}{5}$
Solid Sphere	About a tangential axis		$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$	$\frac{7}{5}$
Spherical Shell	About its diametric axis		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$	$\frac{2}{3}$
Spherical Shell	About a tangential axis		$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$	$\frac{5}{3}$
Long thin rod	About on axis passing through its centre of mass and perpendicular to the rod		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$	
Long thin rod	About an axis passing through its edge and perpendicular to the rod		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$	

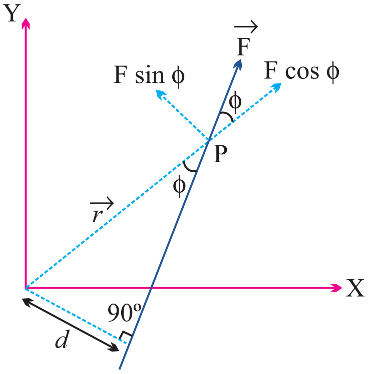
5.14 Torque

If the particle rotating in *xy* plane about the origin under the effect of force $\vec{F}$ and at any instant the position vector of the particle is $\vec{r}$ then,

$$\text{Torque } \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF \sin \phi$$

[where ϕ is the angle between the direction of $\vec{r}$ and $\vec{F}$]



(1) Torque is an axial vector *i.e.*, its direction is always perpendicular to the plane containing vector $\vec{r}$ and $\vec{F}$ in accordance with right hand screw rule. For a given figure the sense of rotation is anti-clockwise so the direction of torque is perpendicular to the plane, outward through the axis of rotation.

i.e., Torque = Force × Perpendicular distance of line of action of force from the axis of rotation.

Torque is also called as moment of force and *d* is called moment or lever arm.

(2) Unit : Newton-metre (M.K.S.) and Dyne-cm (C.G.S.)

(3) Dimension : $[ML^2T^{-2}]$.

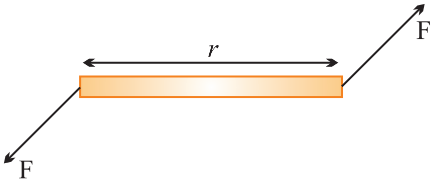
(4) A body is said to be in rotational equilibrium if resultant torque acting on it is zero *i.e.,* $\Sigma \vec{\tau} = 0$.

(5) Torque is the cause of rotatory motion and in rotational motion it plays same role as force plays in translatory motion *i.e.,* torque is rotational analogue of force. This all is evident from the following correspondences between rotatory and translatory motion.

Rotatory Motion	Translatory Motion
$\vec{\tau} = I\vec{\alpha}$	$\vec{F} = m\vec{a}$
$W = \int \vec{\tau} . d\vec{\theta}$	$W = \int \vec{F} . d\vec{s}$
$P = \vec{\tau} . \vec{\omega}$	$P = \vec{F} . \vec{V}$
$\vec{\tau} = \dfrac{d\vec{L}}{dt}$	$\vec{F} = \dfrac{d\vec{P}}{dt}$

5.15 Couple

A couple is defined as combination of two equal but oppositely directed force not acting along the same line. The effect of couple is known by its moment of couple or torque by a couple $\vec{\tau} = \vec{r} \times \vec{F}$.



5.17 Angular Momentum

The moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum. If $\vec{P}$ is the linear momentum of particle and $\vec{r}$ its position vector from the point of rotation then angular momentum.

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = rP \sin\phi \, \hat{n}$$

Angular momentum is an axial vector *i.e.,* always directed perpendicular to the plane of rotation and along the axis of rotation.

(1) S.I. Unit : $kgm^2 s^{-1}$ or J-sec.

(2) Dimension : $[ML^2T^{-2}]$ and it is similar to Planck's constant (h).

(3) Angular momentum = (Linear momentum) × (Perpendicular distance of line of action of force from the axis of rotation)

(4) In vector form $\vec{L} = I\vec{\omega}$.

(5) From $\vec{L} = I\vec{\omega}$, $\therefore \frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha} = \vec{\tau}$

[Rotational analogue of Newton's second law]

(6) If a large torque acts on a particle for a small time then 'angular impulse' of torque is given by $\vec{J} = \int \vec{\tau} dt = \vec{\tau}_{av} \int_{t_1}^{t_2} dt$

$\therefore$ Angular impulse = Change in angular momentum

5.18 Law of Conservation of Angular Momentum

If the net external torque on a particle (or system) is zero then $= \frac{d\vec{L}}{dt} = 0$

i.e.,

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \ldots\ldots\ldots = \text{constant}.$$

Angular momentum of a system (may be particle or body) remains constant if resultant torque acting on it zero.

As L = Iω so if $\vec{\tau} = 0$ then Iω = constant.

5.20 Slipping, Spinning and Rolling

(1) Slipping : When the body slides on a surface without rotation then its motion is called slipping motion.

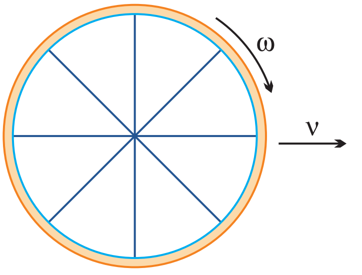
In this condition friction between the body and surface F = 0.

Body possess only translatory kinetic energy $K_T = \frac{1}{2}mv^2$.

(2) Spinning : When the body rotates in such a manner that its axis of rotation does not move then its motion is called spinning motion. In this condition axis of rotation of a body is fixed.

In spinning, body possess only rotatory kinetic energy $K_R = \frac{1}{2}I\omega^2$.

(3) Rolling : If in case of rotational motion of a body about a fixed axis, the axis of rotation also moves, the motion is called combined translatory and rotatory.



Example :

(i) Motion of a wheel of cycle on a road.

(ii) Motion of football rolling on a surface.

In this condition friction between the body and surface F ≠ 0. Body possesses both translational and rotational kinetic energy.

Net kinetic energy = (Translatory + Rotatory) kinetic energy.

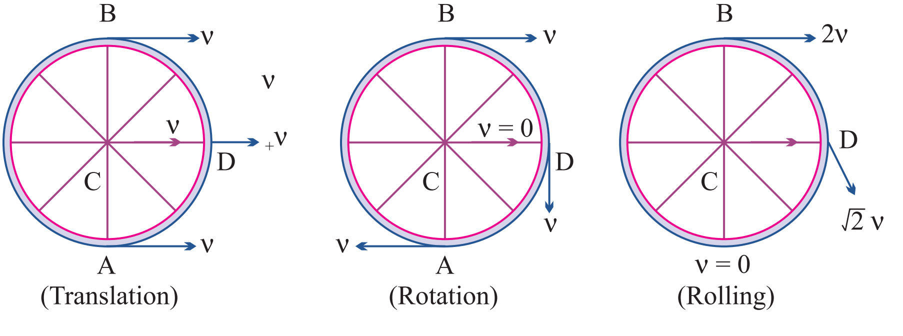
5.21 Rolling Without Slipping

In case of combined translatory and rotatory motion if the object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping.

Friction is responsible for this type of motion but work done or dissipation of energy against friction is zero as there is no relative motion between body and surface at the point of contact.

Rolling motion of a body may be treated as a pure rotation about an axis through point of contact with same angular velocity ω. [$v = R\omega$]

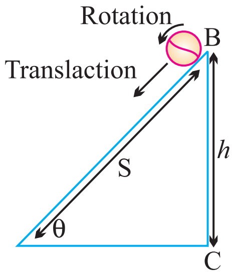
Linear velocity of different points in rolling : In case of rolling, all points of a rigid body have same angular speed but different linear speed. Let A, B, C and D are four points then their velocities are shown in the following figure.



5.22 Rolling on an Inclined Plane

When a body of mass *m* and radius R rolls down on inclined plane of height

'h' and angle of inclination θ, it loses potential energy. However it acquires both linear and angular speeds and hence, gain kinetic energy of translation and that of rotation.



(1) Velocity at the lowest point : $v = \sqrt{\dfrac{2gh}{1+\dfrac{k^2}{R^2}}}$

(2) Acceleration in motion : From equation $v^2 = u^2 + 2as$.

By substituting $u = 0$, $s = \dfrac{h}{\sin\theta}$ and $v = \sqrt{\dfrac{2gh}{1+\dfrac{k^2}{R^2}}}$, we get $a = \dfrac{g\sin\theta}{1+\dfrac{k^2}{R^2}}$.

(3) Time of descent : From equation $v = u + at$

By substituting $u = 0$ and value of v and a from above expressions

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left[1+\frac{k^2}{R^2}\right]}.$$

ROTATIONAL MOTION (1 MARK)

1. About which axis a uniform cube will have minimum moment of inertia ?

2. State the priciple of moments of rotational equilibrium.

3. Find the moment of inertia of a disc of radius R and mass m about an axis in its plane at a distance R/2 from its centre.

4. Can the couple acting on a rigid body produce translator motion ?

5. Which component of linear momentum does not contribute to angular momentum ?

6. A system is in stable equilibrium. What can we say about its potential energy ?

7. Is radius of gyration a constant quantity ?

8. Two solid spheres of the same mass are made of metals of different densities. Which of them has a large moment of inertia about the diameter ?

9. The moment of inertia of two rotating bodies A and B are I_A and I_B ($I_A > I_B$) and their angular momenta are equal. Which one has a greater kinetic energy ?

10. A particle moves on a circular path with decreasing speed. What happens to its angular momentum ?

11. What is the value of instantaneous speed of the point of contact during pure rolling ?

12. Which physical quantity is conserved when a planet revolves around the sun ?

13. What is the value of torque on the planet due to the gravitational force of sun ?

14. If no external torque acts on a body, will its angular velocity be constant ?

15. Why there are two propellers in a helicopter ?

16. A child sits stationary at one end of a long trolley moving uniformly with speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, then what is the effect of the speed of the centre of mass of the (trolley + child) system ?

Rotational motion (2 marks)

17. Show that in the absence of any external force, the velocity of the centre of mass remains constant.

18. State the factors on which the position of centre of mass of a rigid body depends.

19. What is the turning effect of force called for ? On what factors does it depend ?

20. State the factors on which the moment of inertia of a body depends.

21. On what factors does radius of gyration of body depend ?

22. Why the speed of whirl wind in a Tarnado is alarmingly high ?

23. Can a body be in equilibrium while in motion ? If yes, give an example.

24. There is a stick half of which is wooden and half is of steel. (i) it is pivoted at the wooden end and a force is applied at the steel end at right angle to its length (ii) it is pivoted at the steel end and the same force is applied at the wooden end. In which case is the angular acceleration more and why ?

25.If earth contracts to half of its present radius what would be the length of the day at equator ?

26. An internal force can not change the state of motion of centre of mass of a body. How does the internal force of the brakes bring a vehicle to rest ?

27. When does a rigid body said to be in equilibrium ? State the necessary condition for a body to be in equilibrium.

28. How will you distinguish between a hard boiled egg and a raw egg by spinning it on a table top ?

29. Equal torques are applied on a cylinder and a sphere. Both have same mass and radius. Cylinder rotates about its axis and sphere rotates about one of its diameter. Which will acquire greater speed and why ?

30. In which condition a body lying in gravitational field is in stable equilibrium ?

31. Give the physical significance of moment of inertia. Explain the need of fly wheel in Engine.

Rotational motion (3 marks)

32. Derive the three equation of rotational motion

(i) $\omega = \omega_0 + at$ (ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(iii) $\omega^2 = \omega_0^2 + 2\alpha\theta$

Under constant angular acceleration. Here symbols have usual meaning.

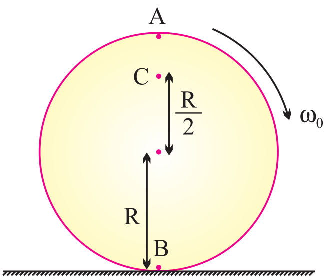
33. Obtain an expression for the work done by a torque. Hence write the expression for power.

34. Prove that the rate of change of angular momentum of a system of particles about a reference point is equal to the net torque acting on the system.

35. Three mass point m_1, m_2, m_3 are located at the vertices of equilateral Δ of side 'a'. What is the moment of inertia of system about an axis along the altitude of Δ passing through m_1 ?

36. Show that moment of a couple does not depend on the point about which moment is calculated.

37. A disc rotating about its axis with angular speed ω_0 is placed lightly (without any linear push) on a perfectly frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in figure. Will the disc roll ?



38. A uniform circular disc of radius R is rolling on a horizontal surface. Determine the tangential velocity (i) at the upper most point (ii) at the centre of mass and (iii) at the point of contact.

39. Explain if the ice on the polar caps of the earth melts, how will it affect the duration of the day ?

40. A solid cylinder rolls down an inclined plane. Its mass is 2 kg and radius 0.1 m. If the height of the include plane is 4 m, what is rotational K.E. when it reaches the foot of the plane ?

41. Find the torque of a force $7i - 3j - 5k$ about the origin which acts on a particle whose position vector is $i + j - k$.

Numericals

42. Three masses 3 kg, 4 kg and 5 kg are located at the corners of an equilateral triangle of side 1 m. Locate the centre of mass of the system.

43. Two particles mass 100 g and 300 g at a given time have velocities $10i - 7j - 3k$ and $7i - 9j + 6k$ ms^{-1} respectively. Determine velocity of COM.

44. From a uniform disc of radius R, a circular disc of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of original disc. Locate the centre of gravity of the resultant flat body.

45. The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds, (i) What is its angular acceleration (assume the acceleration to be uniform) (ii) How many revolutions does the wheel make during this time ?

46. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm, what is the mass of the meter stick ?

47. A solid sphere is rolling on a frictionless plane surface about its axis of symmetry. Find ratio of its rotational energy to its total energy.

48. Calculate the ratio of radii of gyration of a circular ring and a disc of the same radius with respect to the axis passing through their centres and perpendicular to their planes.

49. Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident, (i) What is the angular speed of the two-disc system ? (ii) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy ? Take $\omega_1 \neq \omega_2$.

50. In the HCl molecule, the separating between the nuclei of the two atoms is about 1.27 A (1A = 10^{-10} m). Find the approximate location of the CM of the molecule, given that the chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in all its nucleus.

51. A child stands at the centre of turn table with his two arms out stretched. The turn table is set rotating with an angular speed of 40 rpm. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/3 times the initial value ? Assume that the turn table rotates without friction. (ii) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation.

How do you account for this increase in kinetic energy ?

52. To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine ? Assume that the engine is 100% efficient.

53. A car weighs 1800 kg. The distance between its front and back axles is 1·8 m. Its centre of gravity is 1·05 m behind the front axle. Determine the force exerted by the level ground on each front and back wheel.

ROTATIONAL MOTION (5 MARKS)

54. Prove that the angular momentum of a particle is twice the product of its mass and areal velocity. How does it lead to the Kepler's second law of planetary motion ?

55. Prove the result that the velocity V of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by $v^2 = \dfrac{2gh}{1+\dfrac{k^2}{R^2}}$.

where K = Radius of gyration of body about its symmetry axis, and R is radius of body. The body starts from rest at the top of the plane.

56. (i) Establish the relation between torque and angular acceleration.

Hence define moment of inertia.

(ii) Can a body in translatory motion have angular momentum ? Explain ?

(iii) Establish the relation between angular momentum and moment of inertia for a rigid body.

(iv) Why is it more difficult to revolve a stone by tying it to a longer string than by tying it to a shorter string ?

(v) State the law of conservation of angular momentum and illustrate it with the example of planetary motion.

(vi) A cat is able to land on its feet after a fall. Why ?

57. State the theorem of :

(i) perpendicular axis (ii) parallel axis.

Find the moment of inertia of a rod of mass M and length L about and axis perpendicular to it through one end. Given the moment of inertia about an axis perpendicular to rod and through COM is $\frac{1}{12}ML^2$.

MULTIPLE CHOICE QUESTIONS

58. For which of the following does the center of mass lie out side the body?

(a) Pencil	(b) A Short put
(c) A dice	(d) A bangle

59. When a disc rotates with uniform angular velocity, which of the following is not true?

(a) Some of rotation remains same.
(b) Orientation of the axis of rotation remains same.
(c) The speed of rotation is non-zero and remains same.
(d) The angular acceleration is non-zero and remains same.

60. Two identical particles moves towards each other with velocities 2V and V respectively. The velocity of centre of mass is

(a) V	(b) V/3
(c) V/2	(d) Zero

61. A circular disc of radius R is removed from a bigger circular disc of radius 2R, such that the circumference of the disc coincides. The centre of mass of the new disc is αR from the centre of bigger disc. The value of α is

(a) $\frac{1}{3}$	(b) $\frac{1}{2}$
(c) $\frac{1}{6}$	(d) $\frac{1}{4}$

62. Distance of the centre of mass of a solid uniform cone from it's vertex is Zo. If the radius of it's base is R and it's height is h, the Zo is equal to

(a) $\frac{h^2}{4R}$ (b) $\frac{3h}{4}$

(c) $\frac{5h}{8}$ (d) $\frac{3h^2}{8R}$

63. Angular momentum of the particle rotating with a central force is constant due to

(a) Constant force (b) Constant linear momentum

(c) Constant torque (d) Zero torque

64. Four point masses each of the value m, are placed at the corner of a square ABCD of side l. The moment of inertia of this system about an axis passing through A parallel to B D is

(a) 3 ml^2 (b) ml^2

(c) 2 ml^2 (d) $\sqrt{3}$ ml^2

65. A couple is acting on a two particle system. The resultant motion will be

(a) Purely rotational motion (b) Purely linear motion

(c) Both (a) & (b) (d) Neither (a) nor (b)

66. The dimension of angular momentum are

(a) $[MLT^{-2}]$ (b) $[ML^2T^{-1}]$

(c) $[ML^2T^{-2}]$ (d) $[ML^2T]$

67. Moment of Inertia of an object does not depend up on

(a) Mass of object (b) Mass distribution

(c) Angular velocity (d) Axis of rotation

68. One circular ring and one circular disc both having same mass and radius. The ratio of their moment of inertia about the axis passing through their centres and perpendicular to their planes will be

(a) 1 : 1 (b) 2 : 1

(c) 1 : 2 (d) 4 : 1

69. What is the ratio of the moments of inertia of two rings radii r and nr about an axis perpendicular to their plane and passing through their centres?

(a) 1 : n^2 (b) 1 : n

(c) 1 : 2n (d) n^2 : 1

70. Two rings of radii R and nR made from the same wire have the ratio of moments of inertia about an axis passing through their centres equal to 1:8. The value of n is

(a) 2 (b) $2\sqrt{2}$

(c) 4 (d) ½

71. The moment of inertia of a ring about one of it's diameter is I. What will be the moment of inertia about a tangent parallel to the diameter?

(a) 4 I (b) 2 I

(c) $\frac{3}{2}$ I (d) 3 I

72. A person standing on a rotating disc stretches out his hands, the angular speed will

(a) Increase (b) Decrease

(c) Remains same (d) None of the these

73. A sphere of radius 'r' is rolling without sliding. What is the ratio of rotational kinetic energy and total kinetic energy associated with sphere

(a) $\frac{2}{7}$ (b) $\frac{2}{5}$

(c) 1 (d) $\frac{1}{2}$

74. A solid sphere of radius 'r' is rolling with velocity V on a smooth plane. The total kinetic energy of sphere is

(a) $\frac{7}{10}mv^2$ (b) $\frac{3}{4}mv^2$

(c) $\frac{1}{2}mv^2$ (d) $\frac{1}{4}mv^2$

75. Two bodies have their moment of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momentum will be in the ratio

(a) 1 : 2 (b) $\sqrt{2} : 1$

(c) 2 : 1 (d) $1 : \sqrt{2}$

76. An inclined plane makes on angle of 30° with horizontal. A solid sphere rolling down this inclined plane has a linear acceleration of

(a) $\frac{5g}{14}$ (b) $\frac{2g}{3}$

(c) $\frac{2}{3}$ (d) $\frac{5g}{7}$

77. Planetary motion in the solar system describes

(a) Conservation of kinetic energy

(b) Conservation of linear momentum

(c) Conservation of angular momentum

(d) All of the above.

ANSWERS (ROTATIONAL MOTION) 1 MARK

1. It will be about an axis passing through the centre of the cube and connecting the opposite corners.

2. $\Sigma\overline{\tau} = 0.$

3. $\frac{1}{2}MR^2.$

4. No. It can produce only rotatory motion.

5. Radial Component.

6. P.E. is minimum.

7. No, it changes with the position of axis of rotation.

8. Sphere of small density will have large moment of inertia.

9. $K = \frac{L^2}{2I} \Rightarrow K_B > K_A.$

10. as $\vec{L} = \vec{r} \times m\vec{v}$ *i.e.,* magnitude $\vec{L}$ decreases but direction remains constant.

11. Zero.

12. Angular momentum of planet.

13. Zero.

14. No. $\omega \alpha \frac{I}{I}.$

15. Due to conservation of angular momentum.

16. No change in speed of system as no external force is working.

ANSWERS (2 MARKS)

18. (i) Shape of body

(ii) mass distribution

19. Torque

Factors

(i) Magnitude of force

(ii) Perpendicular distance of force vector from axis of rotation.

20. (i) Mass of body

(ii) Size and shape of body

(iii) Mass distribution w.r.t. axis of rotation

(iv) Position and orientation of rotational axis

21. Mass distribution.

22. In this, air from nearly regions get concentrated in a small space, so I $\downarrow$ considerably. Since I.W = constant so W$\uparrow$ so high.

23. Yes, if body has no linear and angular acceleration. Hence a body in uniform straight line motion will be in equilibrium.

24.
$$I \text{ (first case)} > I \text{ (Second case)}$$
$$\because \quad \tau = I\alpha$$
$$\Rightarrow \quad \alpha \text{ (first case)} < \alpha \text{ (second case)}$$

25.
$$I_1 = \frac{2}{5}MR^2 \Rightarrow I_2 = \frac{2}{5}M\left(\frac{R}{2}\right)^2 \Rightarrow I_2 = \frac{I_1}{4}$$
$$L = I_1\omega_1 = I_2\omega_2$$

or
$$I\left(\frac{2\pi}{T_1}\right) = \frac{1}{4}\left(\frac{2\pi}{T_2}\right)$$

or
$$T_2 = \frac{T_1}{4} = \frac{24}{4} = 6 \text{ hours}$$

26. In this case the force which bring the vehicle to rest is friction, and it is an external force.

27. For translation equilibrium

$$\Sigma \vec{F}_{ext} = 0$$

For rotational equilibrium

$$\Sigma \vec{\tau}_{ext} = 0$$

28. For same external torque, angular acceleration of raw egg will be small than that of Hard boiled egg.

29.
$$\tau = I\,\alpha,\ \alpha = \frac{\tau}{I}$$

α in cylinder, $\alpha_C = \frac{\tau}{I_C}$

α in sphere, $\alpha_S = \frac{\tau}{I_S}$

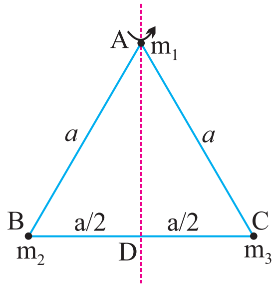
$$\frac{\alpha_C}{\alpha_S} = \frac{I_S}{I_C} = \frac{\frac{2}{5}MR^2}{MR^2} = \frac{2}{5}$$

30. When vertical line through centre of gravity passes through the base of the body.

31. It plays the same role in rotatory motion as the mass does in translatory motion.

ANSWERS (3 MARKS)

35.



$$I = \sum_{i=1}^{n} m_i r_i^2$$

$$= m_1 \times 0 + m_2 \times (BD)^2 + m_3 \times (DC)^2$$

$$= 0 + m_2\left(\frac{a}{2}\right)^2 + m_3\left(\frac{a}{2}\right)^2$$

$$I = \frac{1}{4}(m_2 + m_3)a^2$$

37. For A $V_A = R\omega_0$ in forward direction

For B = $V_B = R\omega_0$ in backward direction

For C $V_C = \frac{R}{2}\omega_0$ in forward direction disc will not roll.

41. $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 7 & -3 & -5 \end{vmatrix} = -8\hat{i} - 2\hat{j} - 10\hat{k}$

ANSWERS (NUMERICALS)

42. $(x, y) = (0.54 \text{ m}, 0.36 \text{ m})$

43. Velocity of COM = $\frac{31i - 34j + 15k}{2} \text{ms}^{-1}$.

44. COM of resulting portion lies at R/6 from the centre of the original disc in a direction opposite to the centre of the cut out portion.

45. $\alpha = 4\pi \text{ rad s}^{-1}$

$n = 576$

46. $m = 66{\cdot}0$ gm.

47.

$$\text{Rot. K.E.} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{2}{5}MR^2 \times \frac{V^2}{R^2}$$

$$\left(\text{as } \omega = \frac{V}{R}, I = \frac{2}{5}MR^2\right)$$

$$= \frac{1}{5}mv^2$$

Total energy = Translational K.E. + Rot. K.E.

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

$$\therefore \quad \frac{\text{Rot. K.E.}}{\text{Total Energy}} = \frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2} = \frac{2}{7}.$$

48. 2 : 1

49. (i) Let *w* be the angular speed of the two-disc system. Then by conservation of angular momentum.

$$(I_1 + I_2)\,\omega = I_1\omega_1 + I_2\omega_2$$

or $$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

(ii) Initial K.E. of the two discs.

$$K_1 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_1\omega_2{}^2$$

Final K.E. of the two disc system.

$$K_2 = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

Loss in K.E.

$$= K_1 - K_2 = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2) - \frac{1}{2(I_1 + I_2)}(I_1\omega_1^2 + I_2\omega_2^2)$$

$$= \frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2 = \text{a positive quantity } [\because\ \omega_1 \neq \omega_2]$$

Hence there is a loss of rotational K.E. which appears as heat.

When the two discs are brought together, work is done against friction between the two discs.

50. As shown in Fig. suppose the H nucleus is located at the origin. Then

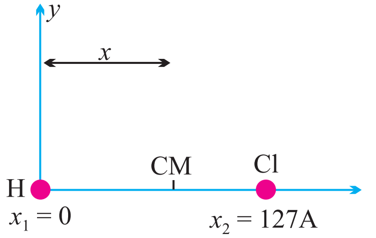
$x_1 = 0$, $x_2 = 1.27$ Å, $m_1 = 1$, $m_2 = 35.5$

The position of the CM of HCl molecule is

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$\frac{1\times 0 + 35.5\times 1.27}{1 + 35.5} = 1.239\ \text{Å}$$

Thus the CM of HCl is located on the line joining H and Cl nuclei at a distance of 1.235 Å from the H nucleus.



51. Here $\omega_1 = 40$ rpm, $I_2 = \frac{2}{5} I_1$

By the principle of conservation of angular momentum,

$$I_1 \omega_1 = I_2 \omega_2 \text{ or } I_1 \times 40 = \frac{2}{5} I_1 \omega_2 \text{ or } \omega_2 = 100 \text{ rpm.}$$

(ii) Initial kinetic energy of rotation

$$\frac{2}{5} I_1 \omega_1^2 = \frac{2}{5} I_1 (40)^2 = 800\, I_1$$

New kinetic energy of rotation

$$\frac{2}{5} I_2 \omega_2^2 = \frac{1}{2} \times \frac{2}{3} I_1 (100)^2 = 2000\, I_1$$

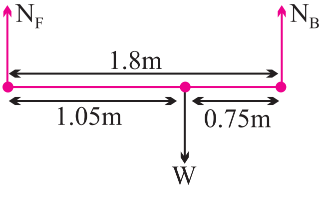
$$\frac{\text{New K.E.}}{\text{Initial K.E.}} = \frac{2000 I_1}{800 I_1} = 2.5$$

Thus the child's new kinetic energy of rotation is 2.5 times its initial kinetic energy of rotation. This increase in kinetic energy is due to the internal energy of the child which he uses in folding his hands back from the out stretched position.

52. Here $\omega = 200$ rad s^{-1}, $\tau = 180$ Nm

$\because$ Power, $P = \tau\, \omega = 180 \times 200 = 36{,}000$ W = 36 kW.

53.



For translation equilibrium of car

$$N_F + N_B = W = 1800 \times 9{\cdot}8 = 17640 \text{ N}$$

For rotational equilibrium of car

$$1{\cdot}05\, N_F = 0{\cdot}75\, N_B$$

$$1{\cdot}05\ N_F = 0{\cdot}75\ (17640 - N_F)$$
$$1{\cdot}8\ N_F = 13230$$
$$N_F = 13230/1{\cdot}8 = 7350\ N$$
$$N_B = 17640 - 7350 = 10290\ N$$

Force on each front wheel= $\dfrac{7350}{2} = 3675$ N

Force on each back wheel= $\dfrac{10290}{2} = 5145$ N

ANSWERS (5 MARKS)

56. (ii) Yes, a body in translatory motion shall have angular momentum unless fixed point about which angular momentum is taken lies on the line of motion of body

$\therefore$ $$|\overline{L}| = rp \sin\theta$$
$$= 0 \text{ only when } \theta = 0^\circ \text{ or } 180^\circ$$

(iv) MI of stone $I = ml^2$ ($l \rightarrow$ length of string)

$$\tau = I\alpha$$
$$\alpha = \frac{\tau}{I} = \frac{\tau}{ml^2}$$

if l is large α is very small

$\therefore$ more difficult to revolve.

Answer (MCQ) Key :

58. (d)	59. (d)	60. (c)	61. (a)	62. (b)	63. (d)
64. (a)	65. (a)	66.	67.	68. (b)	69. (a)
70. (a)	71. (d)	72. (b)	73. (a)	74. (a)	75. (d)
76. (a)	77. (c)				

HINTS AND SOLUTION (MCQ)

58. (d) In bangle centre of mass lies at it's centre.

59. (d) $\alpha = \frac{dw}{dt}$, given w = constant

Hence $\alpha = 0$

60. (c) $V_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{m(2v) + m(-v)}{2m} = \frac{V}{2}$

61. (a) Mass of original disc = m

Mass of disc removed, $m_1 = \frac{m}{4}$

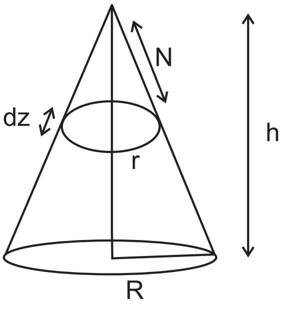
Mass of remaining disc $= \frac{3m}{4}$

Mass m_1 and m_2 are concentrated at O_1 and O_2 respectively and O is their centre of mass.

Moment of m_1 about O = moment of m_2 about O.

$$\frac{m}{4} \times R = \frac{3m}{R} \times \alpha R \quad \Rightarrow \quad \alpha = \frac{1}{3}$$

62. Mass of elementary disc,



$dm = \pi r^2\ dze$

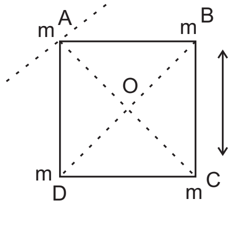
$$\frac{r}{R} = \frac{z}{h} \Rightarrow r = \frac{Rz}{h}$$

$$Z_O = \frac{\int z dm}{\int dm} = \frac{\int_0^h \pi r^2 dzez}{\frac{1}{3}\pi R^2 he} = \frac{3}{4}h$$

63. (d) Torque due to central force is zero

$$T = \frac{dL}{dt} = 0 \Rightarrow L = \text{constant.}$$

64. (a) $AC = BD = \sqrt{2}\,\ell$



$$I_{BD} = m\,(AO)^2 + m\,(CO)^2 = ml^2$$

65. (a) Net force for a couple is zero.

So, couple produces only rotational motion.

66. (b) $\vec{L} = \vec{r} \times \vec{p}$ $[L] = [M\,L^2\,T^{-1}]$

67. (c) M.I. does not depent up on angular velocity.

68. (b) $\frac{I_{ring}}{I_{disc}} = \frac{mR^2}{\frac{1}{2}mR^2} = 2 : 1$

69. (a) $\frac{I_1}{I_2} = \frac{Mr^2}{m(nr)^2} = 1 : n^2$

70. (a) As radius of round ring is n times, length and hence mass of wire is also n times

$$\frac{I_1}{I_2} = \frac{mR^2}{nM(nR)^2} = \frac{1}{n^3} = \frac{1}{8} \Rightarrow \boxed{n = 2}$$

71. (d) $I_T = I + mR^2 = \frac{1}{2}mR^2 + mR^2$

$$= 3 \times \frac{1}{2}mR^2 = 3\,I$$

72. (b) As person stretches his hands outward, hence moment of inertia, I = increases

L = Iw = constant, So w decreases.

73. (a)

$$E_T = E_{trans} + E_{rot} = \frac{1}{2}\ mV^2 + \frac{1}{2}\ Iw^2$$

$$I = \frac{2}{5}mr^2,\ E_T = \frac{7}{10}\ mV^2$$

$$\frac{E_{rot}}{E_{trans}} = \frac{\frac{1}{5}\ mV^2}{\frac{7}{10}\ mV^2} = \frac{2}{7}$$

74. (a) $E_T = E_{trans} + E_{rot} = \frac{7}{10}\ mV^2$

75. (d) Rotational K.E. = $\frac{1}{2}Iw^2$

$$\frac{1}{2}I_1 w_1^2 = \frac{1}{2}I_2 w_2^2 \Rightarrow w_2 = \frac{w_1}{\sqrt{2}}$$

$$\frac{L_1}{L_2} = \frac{1}{\sqrt{2}}$$

76. (d) $a = \frac{5}{7}g \sin 30° = \frac{5g}{dt}$

77. (c) Feet = 0, So, T = 0 $= \frac{dL}{dt}$

$\Rightarrow \vec{L} =$ constant.

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